**Joint Distribution**

Similar to the one-dimensional situation, we can denote the range space of by:

is a discrete two-dimensional random variable if the number of possible values of are finite or countable. That is, the possible values of may be represented by

is a continuous two-dimensional random variable if the possible values of can assume any value in some region of the Euclidean space

**The joint probability (mass) function** of a discrete random variable is

and

for x,y in the range of x,y

for x,y not in the range of x,y

**The joint probability density function** of a continuous random variable is

**Marginal Probability Distribution** of x

Discrete:

Continuous:

**Conditional Probability Distribution** of Y given X = x:

*(the distribution of Y given that the random variable X is observed to take the value x)*

*(the distribution of X given that the random variable Y is observed to take the value y)*

Random variables X and Y are independent if and only if for any x and y,

**Properties of Independent Random Variables**

If X and Y are independent random variables, the following properties hold:

1. For any arbitrary subsets A and B of R, the events X ∈ A and Y ∈ B are independent events in S. Thus, In particular, for any real numbers x, and y, we have
2. Independence is connected with conditional distribution

**Expectation**

Consider a 2 variable function

Remember If  is a discrete random variable,

If  is a continuous random variable,

If $ the expectation leads to the covariance of X and Y.

**Covariance**

If is a discrete random variable,

If is a continuous random variable,

**Random Variables**

**Probability mass function**

For a discrete random variable X, define

n , is the probability function or probability mass function.

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**Probability density function**

For a continuous random variable X, define

Notice that

**Cumulative distribution function**

For a random variable X,

**CDF of discrete random variable**

a- is the largest smaller than a. Notice that

**CDF of continuous random variable**

**Expectation**

For a discrete random variable:

For a continuous random variable:

Properties:

**Variance**

Properties:

**Basic Concepts of Probability**

A **statistical experiment** is any procedure that produces data or observations. The **sample space**, denoted by S, is the set of all possible outcomes of a statistical experiment. The sample space depends on the problem of interest! A **sample point** is an outcome (element) in the sample space. An **event** is a subset of the sample space.

Conditional Probability

A statistical experiment is any procedure that produces data or observations. The sample space, denoted by S, is the set of all possible outcomes of a statistical experiment. The sample space depends on the problem of interest! A sample point is an outcome (element) in the sample space. An event is a subset of the sample space.

Multiplication rule

Inversion probability rule

**Independence**

If A and B are independent:

**Law of total probability**

For a special case with any events A and B:

**Bayes Theorem**

We can also see, as an extension of the inversion probability rule

Things to note: